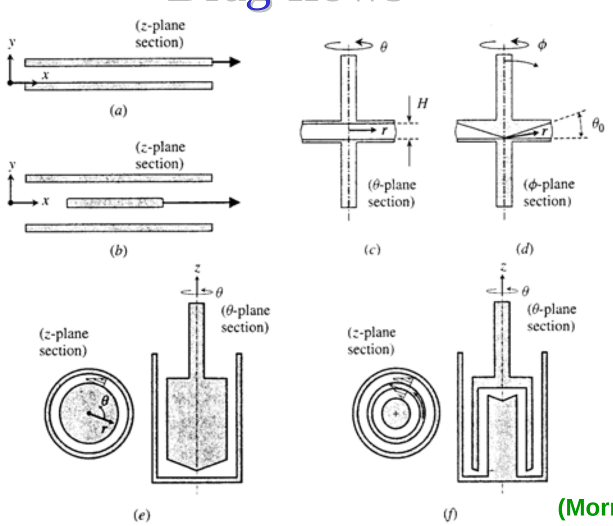
Drag flows

- Majority of commercial rheometers → rotating devices
- Three possible geometries:
  - Coaxial cylinders (Couette)
  - Cone and plate
  - Concentric disks
- Enable characterization of viscosity and other important rheological functions
- Commercial instruments now available with computer control and data analysis software packages

1

Drag flows

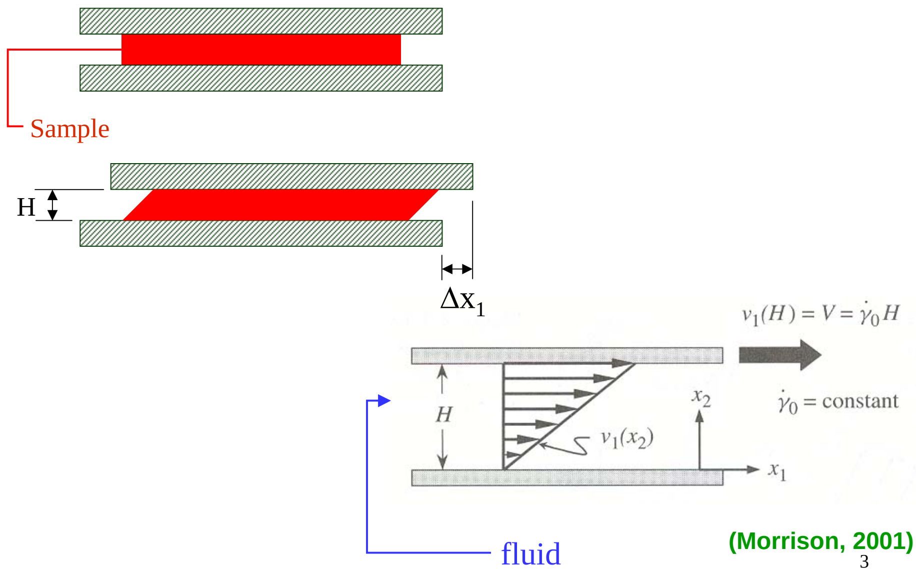


(Morrison, 2001)

Figure 4.4 Geometries used to produce shear flow in commercial and research rheometers. (*a*) Rectilinear parallel plate. (*b*) Rectilinear double parallel plate. (*c*) Torsional parallel plate or parallel disk. (*d*) Torsional cone and plate. (*e*) Couette or cup and bob. (*f*) Double-walled Couette.

2

Parallel plate (sliding) rheometers



(Morrison, 2001)

3

Parallel plate (sliding) rheometers

- The total strain imposed is limited by the length of the rheometer
- The gap *h* should be made as small as possible

$$\gamma = \frac{\Delta x}{\Delta y} = \frac{\Delta x}{h}$$

$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{1}{h}\frac{d\Delta x}{dt} = \frac{V}{h}$$

4

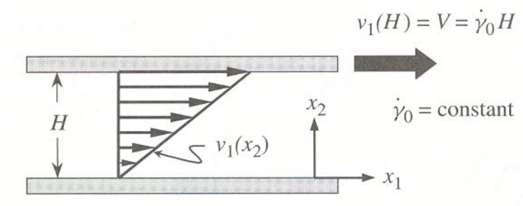
Parallel plate (sliding) rheometers

$v_1 \neq 0 = f(x_2)$

$v_2 = v_3 = 0$

$v_1 = Vx_2/H$

$\sigma_{x1x2} = F/A$



For a Newtonian fluid: $\sigma_{x1x2} = \eta V/H$

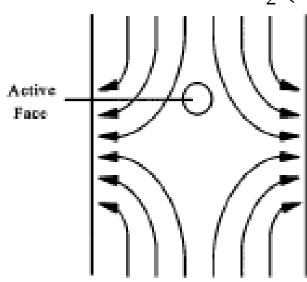
$A = W \times L$

Total force on the plate: $F = WL\eta V/H$

5

Parallel plate (sliding) rheometers

- Sources of errors:
  - End and edge effects → use of a pressure-transducer to measure the stress locally
  - Pumping of the fluid due to N_2 (secondary flow)

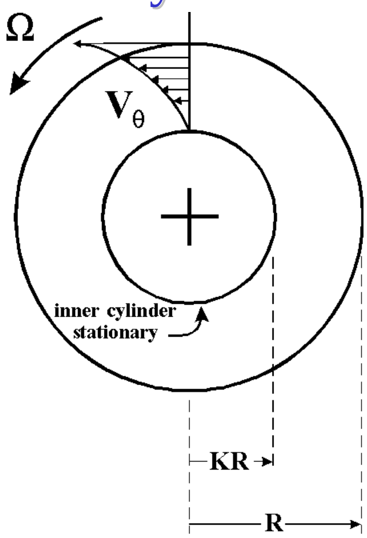


Reimers and Dealy (1996)

FIG. 4. Top view of the sliding plate rheometer illustrating the flow direction of the secondary flow.

6

3.2 Coaxial-cylinder rheometers

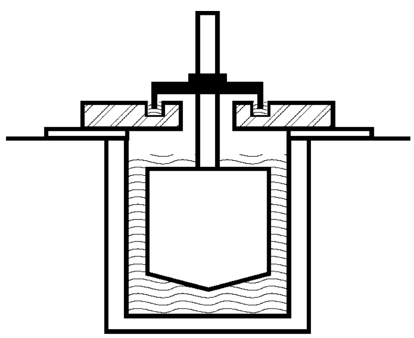


Couette flow

7

Coaxial-cylinder rheometers

- Probably the first rotating devices to measure viscosity
- Named after Maurice Couette (1890), who invented it
- Assumed that the resistance to flow occurs in the small gap
- Used to determine the viscosity of low-viscosity fluids

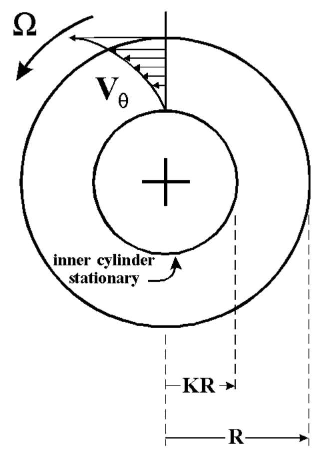


Couette geometry for Bohlin VOR rheometer with evaporation control

8

Calculation of viscosity

- Two coaxial cylinders of length L and radii R and KR
- One cylinder rotates at a known angular velocity Ω, the other one is stationary
- Measure of the torque exerted on the outer cylinder
- The flow is viscometric in cylindrical coordinates
- The fluid moves in circular patterns



9

Calculation of viscosity

- Assumptions:
  - steady-state
  - isothermal flow
  - end effects neglected (unidirectional flow)

$$V_\theta = V_\theta(r)$$

$$V_r = V_z = 0$$

The θ-component of the equation of motion reduces to

$$0 = -\frac{1}{r^2}\frac{d}{dr}(r^2\sigma_{r\theta})$$

Integrated gives $\sigma_{r\theta} = C_1 / r^2$

10

Calculation of viscosity

- To evaluate the constant C_1, we need a rheological model. Assumption of power-law model:

$$\sigma_{r\theta} = -m\left|r\frac{d}{dr}\left(\frac{V_\theta}{r}\right)\right|^n$$

- Shear rate in cylindrical coordinates is given by:

$$\dot{\gamma}_{r\theta} = r\frac{d}{dr}\left(\frac{V_\theta}{r}\right) \qquad \text{(r}\theta\text{-component)}$$

- Combining previous equations and integrating:

$$\frac{V_\theta}{r} = \frac{n}{2}\left|-\frac{C_1}{mR^2}\right|^{1/n}\left[\left(\frac{R}{r}\right)^{2/n} + C_2\right]$$

11

Calculation of viscosity

- The two constants C_1 and C_2 are evaluated using a no-slip assumption:

at $r = KR$, $V_\theta = 0$

at $r = R$, $V_\theta = \Omega R$.

The velocity profile is therefore:

$$\frac{V_\theta}{r} = \frac{\Omega}{1-(1/K)^{2/n}}\left[\left(\frac{R}{r}\right)^{2/n} - \left(\frac{1}{K}\right)^{2/n}\right]$$

And the rθ-component of the rate-of-strain tensor is:

$$\dot{\gamma}_{r\theta} = \left(\frac{R}{r}\right)^{2/n}\left[\frac{2\Omega}{n\left[(1/K)^{2/n}-1\right]}\right]$$

12

Calculation of viscosity

- For a small gap, the shear rate is almost constant throughout the gap. In the limit of K→1, it is constant:

$$\dot{\gamma}_{r\theta} = \left| \frac{\Omega}{1-K} \right|$$

- The shear stress is given by:

$$\sigma_{r\theta} = -m\left(\frac{R}{r}\right)^2 \left| \frac{2\Omega}{n\left[(1/K)^{2/n} - 1\right]} \right|^n$$

- The torque exerted on the outer cylinder is:

$$T = 2\pi R^2 L\, m \left| \frac{2\Omega}{n\left[(1/K)^{2/n} - 1\right]} \right|^n$$ (n = 1 for a Newtonian fluid)

13

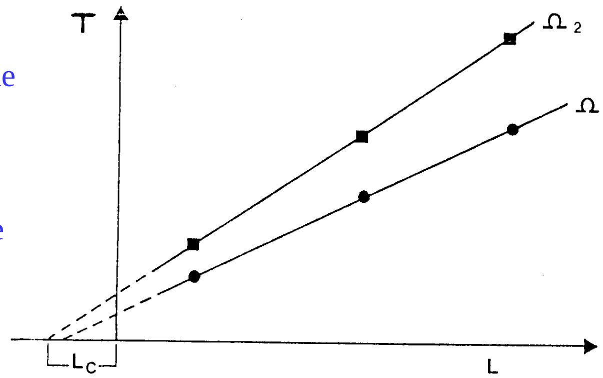
End effect corrections

- Can be minimized by using a small gap geometry or a more sophisticated geometry
- If not, simple method exists to detect and correct for end effects:
  - Experiments can be conducted using standard or Newtonian fluids with a series of constant diameter cylinders with different length L (or different heights of liquids)
  - We plot the torque as a function of L for various values of the rotational speed

14

End effect corrections

The intercept with the x-axis gives L_c, the correction term, equivalent to an additional length due to end effects



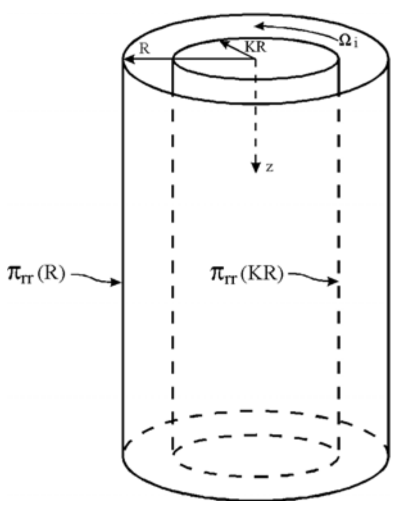
The shear stress is then corrected:

$$-\sigma_{r\theta}(KR) = \frac{T}{2\pi K^2 R^2 (L + L_c)}$$

15

Normal stress determination

- Couette geometry can be used to measure normal stress differences
- Assume:
  - that the flow approximates simple shearing
  - that the only non-zero stress components are $\sigma_{r\theta}(r)$, $\sigma_{rr}(r)$, $\sigma_{\theta\theta}(r)$ and $\sigma_{zz}(r)$



16

Normal stress determination

- r-component of the equation of motion:

$$-\rho \frac{V_\theta^2}{r} = -\frac{\partial P}{\partial r} - \left(\frac{1}{r} \frac{d}{dr} \left(r\sigma_{rr} \right) - \sigma_{\theta\theta} / r \right)$$

- Written for any position z as:

$$\frac{d}{dr} \left(P + \sigma_{rr} \right) = \frac{d\pi_{rr}}{dr} = \rho \frac{V_\theta^2}{r} + \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r}$$

- Integrating with respect to r from KR to R:

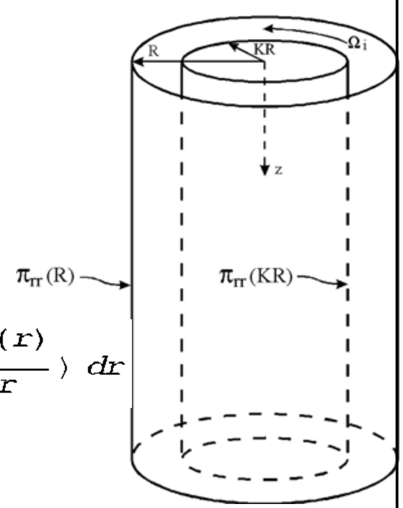
$$\pi_{rr}(R) - \pi_{rr}(KR) = \int_{KR}^{R} (\rho \frac{V_\theta^2}{r} + \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r}) \, dr$$

17

Normal stress determination

- π_{rr}(R) and π_{rr}(KR) are the total pressures measured at the outer and inner cylinder walls respectively
- From the definition of normal stress difference:

$$\pi_{rr}(R) - \pi_{rr}(KR) = \int_{KR}^{R} (\rho \frac{V_\theta^2}{r} - \frac{N_1(r)}{r}) \, dr$$



18

Normal stress determination

- For a small gap (K → 1):

$$\dot{\gamma}_{r\theta} \approx -\frac{\Omega_i KR}{R-KR} = -\frac{\Omega_i}{1-K}$$

- The velocity profile is:

$$V_\theta \approx \frac{\Omega_i KR}{1-K}\left(1-\frac{r}{R}\right)$$

- Integrating the equation for the total stress gives:

$$\pi_{rr}(R)-\pi_{rr}(KR)=\rho\frac{\Omega_i^2(KR)^2}{(1-K)^2}\left[\frac{1}{2}(1-K^2)-2(1-K)+\ln\left(\frac{1}{K}\right)\right]-N_1\ln\left(\frac{1}{K}\right)$$

19

Normal stress determination

- For very small gaps:
  - $\ln(1/K) \approx 1-K$
  - $1-K^2 = (1+K)(1-K) \approx 2(1-K)$

- The inertial (first) term is therefore negligible:

$$\pi_{rr}(R) - \pi_{rr}(KR) \approx (K-1)N_1 = (K-1)\,\psi_1\dot{\gamma}_{r\theta}^2$$

with ψ_1 the primary normal stress coefficient.

20

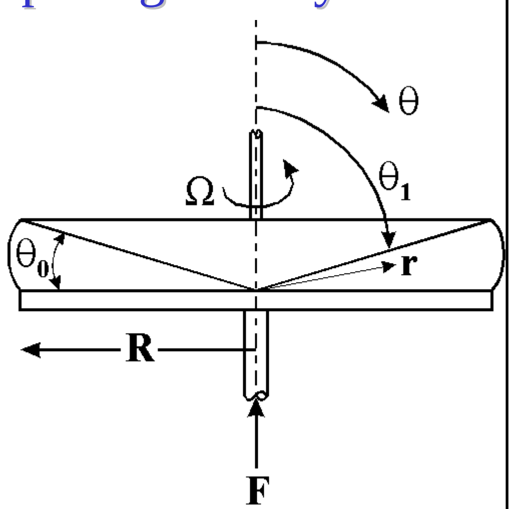
Coaxial-cylinder rheometers

- Sources of errors:
  - Weissenberg effect (rod climbing) for polymeric liquids (previous demonstration on normal forces)
  - Difficulty of maintaining concentricity, mostly when small gaps are used and at elevated temperatures
- No commercial instruments offer the option of measuring the primary normal stress difference with the Couette geometry
  - Technically very difficult to measure pressures at a curved wall surface of a rotating device

21

3.3 Cone-and-plate geometry

- Most popular geometry for rheological measurements of viscoelastic fluids
- Fluid sample is placed between a plate and a cone of the same radius and a very small angle
- The flow is viscometric in spherical coordinates



22

Cone-and-plate geometry

- For a very small angle, the velocity profile is linear with respect to the θ position in the gap:

$$\frac{V_\phi}{r} = \Omega\left[\frac{(\pi/2)-\theta}{\theta_0}\right]$$

- Ω is the imposed angular rotational speed of the cone (or plate)
- The shear rate is the θϕ-component of the rate-of-deformation tensor:

$$\dot{\gamma} = \dot{\gamma}_{\theta\phi} = \frac{\sin\theta}{r}\left|\frac{\partial}{\partial\theta}\left(\frac{V_\phi}{\sin\theta}\right)\right| \approx -\frac{\Omega}{\theta_0} \qquad (\sin\theta \approx 1)$$

23

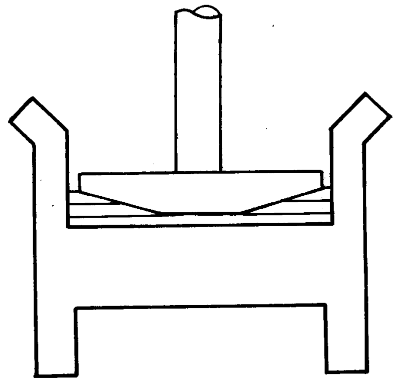
Cone-and-plate geometry

- Main advantages:
  - Shear rate is constant → no flow kinematics or rheological model assumptions needed
  - Very small samples required (< 0.5 mL)
  - Good heat transfer and T° control
  - End effects negligible for low rotational speed and appropriate quantity of fluid in the gap

24

Cone-and-plate geometry

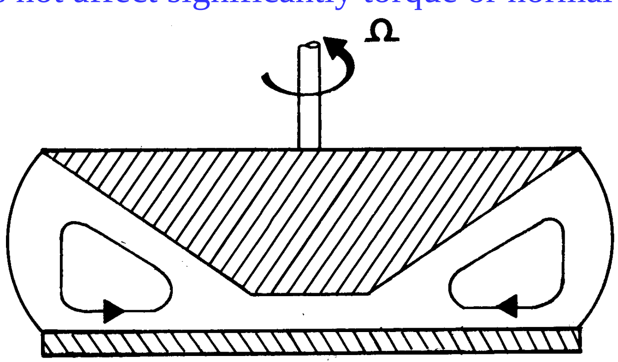
- Disadvantages:
  - Limited to very low shear rate for polymer melts. A cup can be used for low viscosity fluids at large shear rates
  - Difficult to eliminate evaporation
  - Erroneous results can be obtained for multiphase systems



25

Truncated cones

- Easier to set the gap
- Truncation restricted to a small central portion → does not affect significantly torque or normal force



26

Viscosity determination

- Shear rate and shear stress constant across the gap
- Viscosity easily obtained from the torque:

$$T = 2\pi \int_0^R \sigma_{\theta\phi} r^2 dr = \frac{2}{3}\pi R^3 \sigma_{\theta\phi}$$

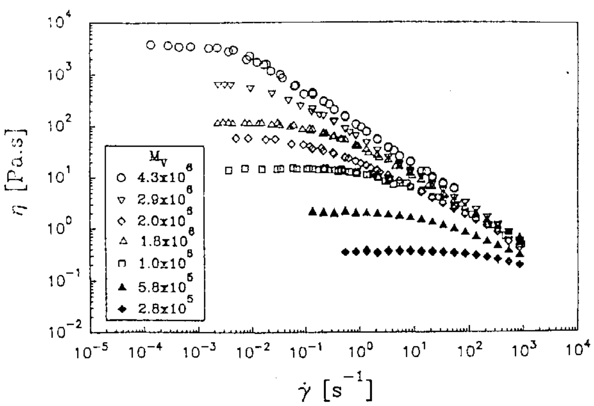
$$\sigma_{\theta\phi} = -\eta \dot{\gamma}_{\theta\phi} = -\eta \dot{\gamma}$$

$$\eta = \frac{3\theta_0 T}{2\pi R^3 \Omega}$$

Viscosity proportional to the torque, and inversely proportional to the rotational speed (in rad/s)

27

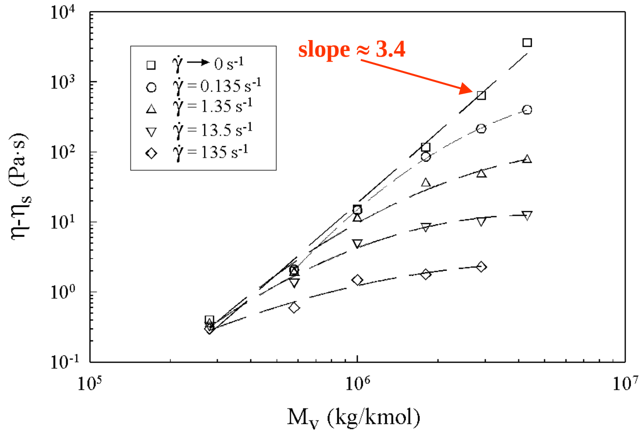
Steady shear data obtained with cone and plate geometry



3% wt PEO solutions in water and glycerine (25°C) (Ortiz, 1992)

28

Steady shear data obtained with cone and plate geometry



3% wt PEO solutions in water and glycerine (25°C) (Ortiz, 1992)

29