Shear-free or elongational flows

These flows have zero off-diagonal components in $\underline{\underline{\dot{\gamma}}}$ or $\underline{\underline{\sigma}}$.

Three simple shear-free flows:

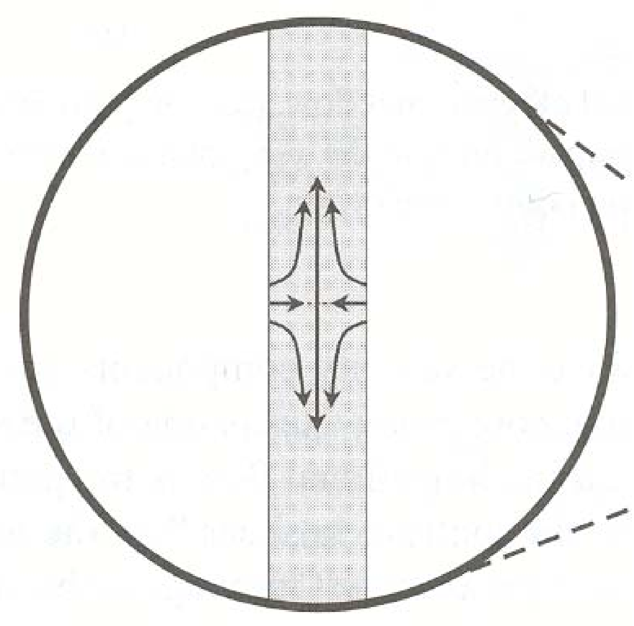
1) Uniaxial elongational flow
2) Biaxial stretching flow
3) Planar elongational flow

Uniaxial elongational flow

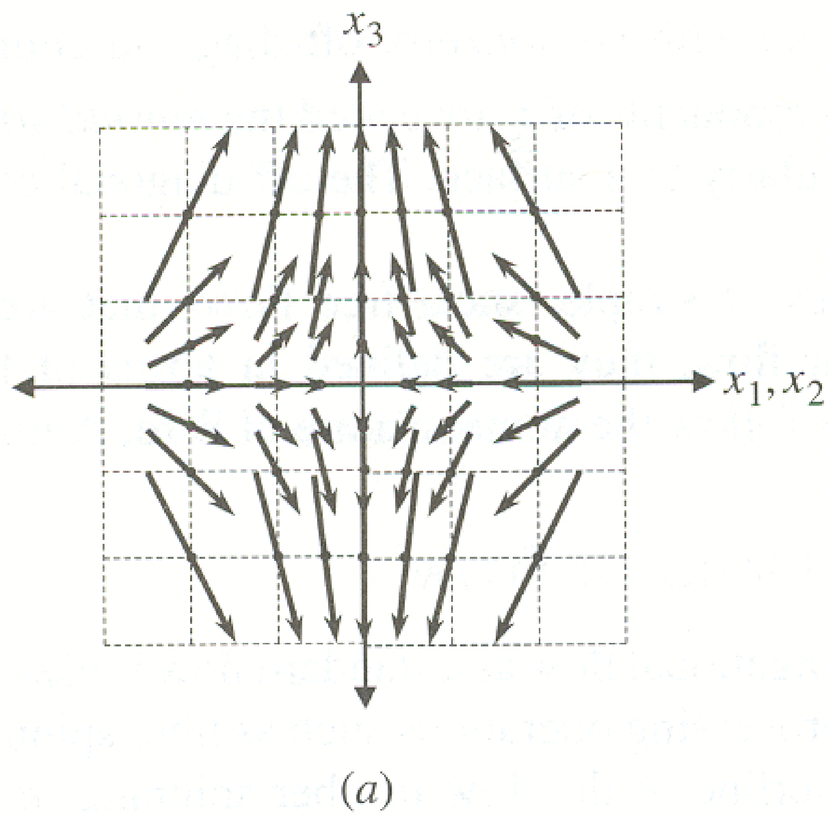
Strong stretch in the 3-direction and contraction occurring equally in the 1-and 2-directions

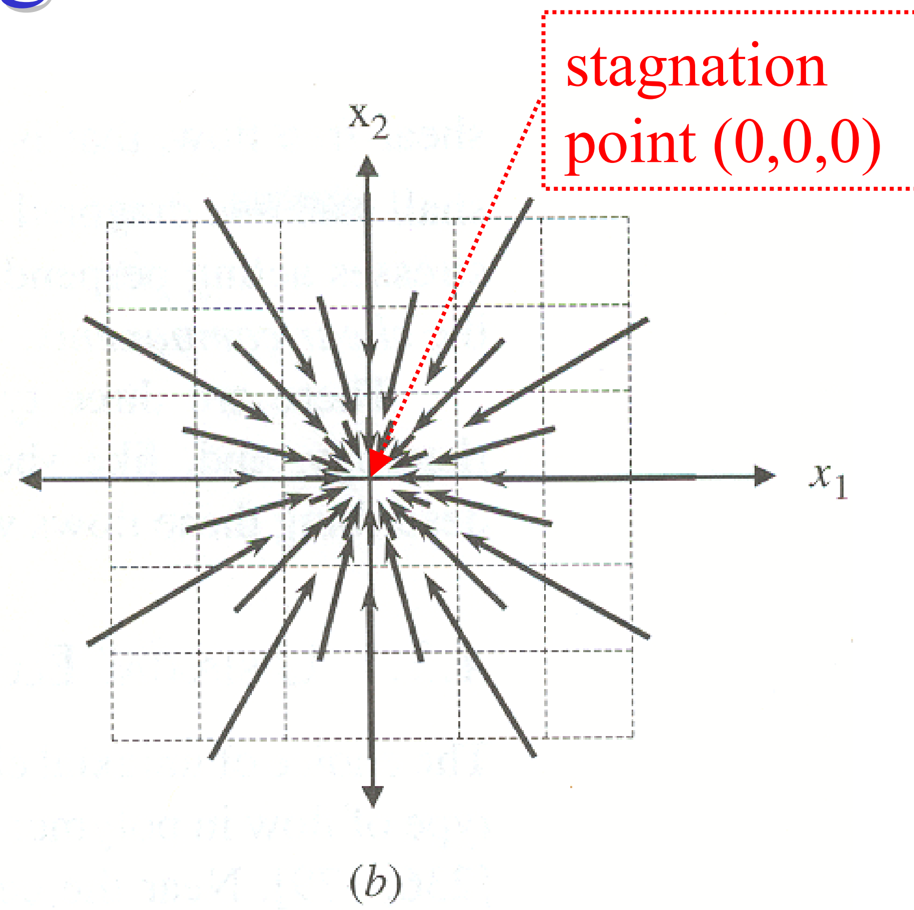
$$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123} = \begin{pmatrix} -\frac{\dot{\varepsilon}(t)}{2} x_1 \\ -\frac{\dot{\varepsilon}(t)}{2} x_2 \\ \dot{\varepsilon}(t) x_3 \end{pmatrix}_{123}$$

where $\dot{\varepsilon}(t) > 0$



Uniaxial elongational flow





Velocity field in the x_3x_1 and x_3x_2 planes

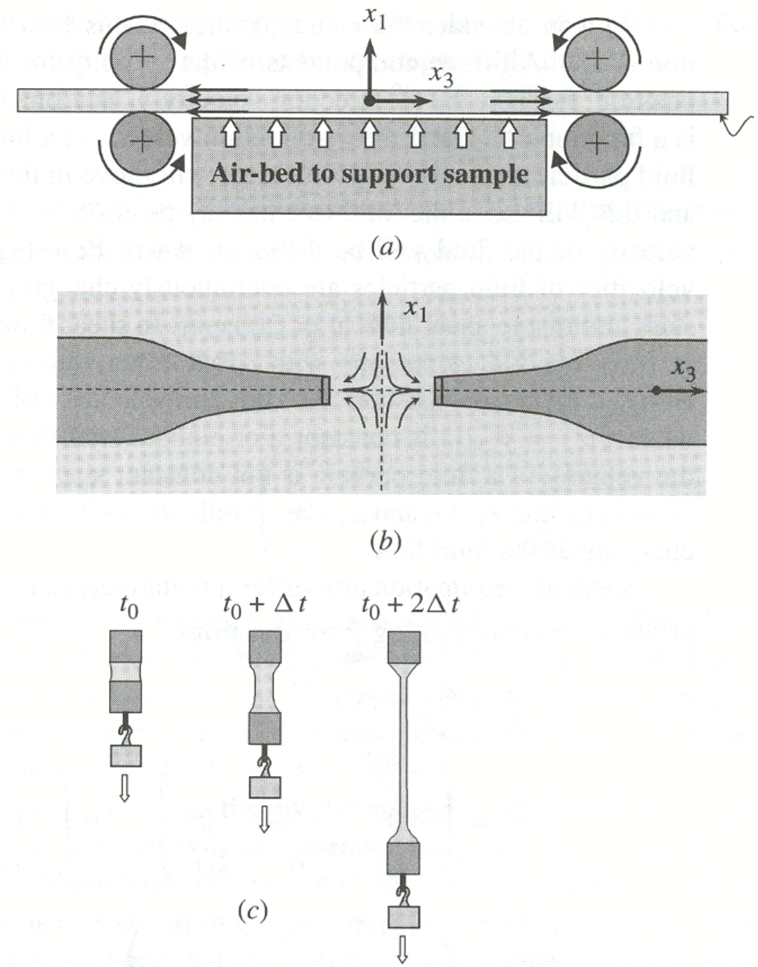
Velocity field in the x_1x_2 plane

Geometries used to produce uniaxial extensional flows

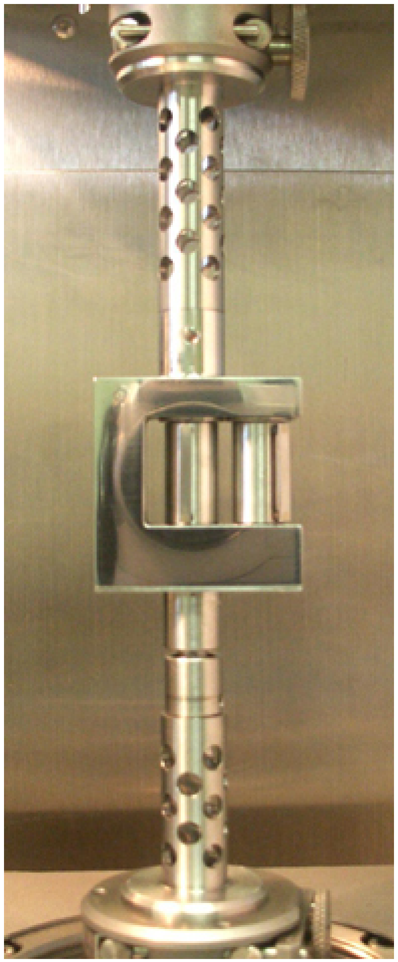
Pulling device (Meissner rheometer)

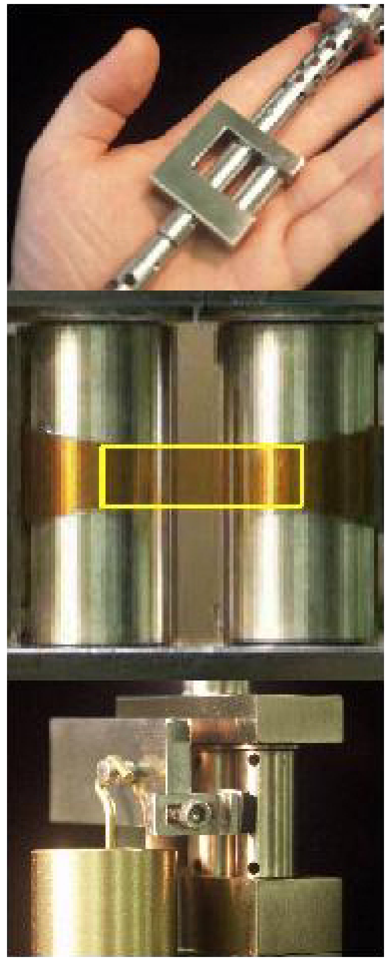
Opposed-nozzle suction device

Filament stretching



SER Uniaxial extension rheometer





- True uniform extensional deformation
- Polymeric materials
- Solids tensile testing
- Tear testing
- Peel testing
- Friction testing

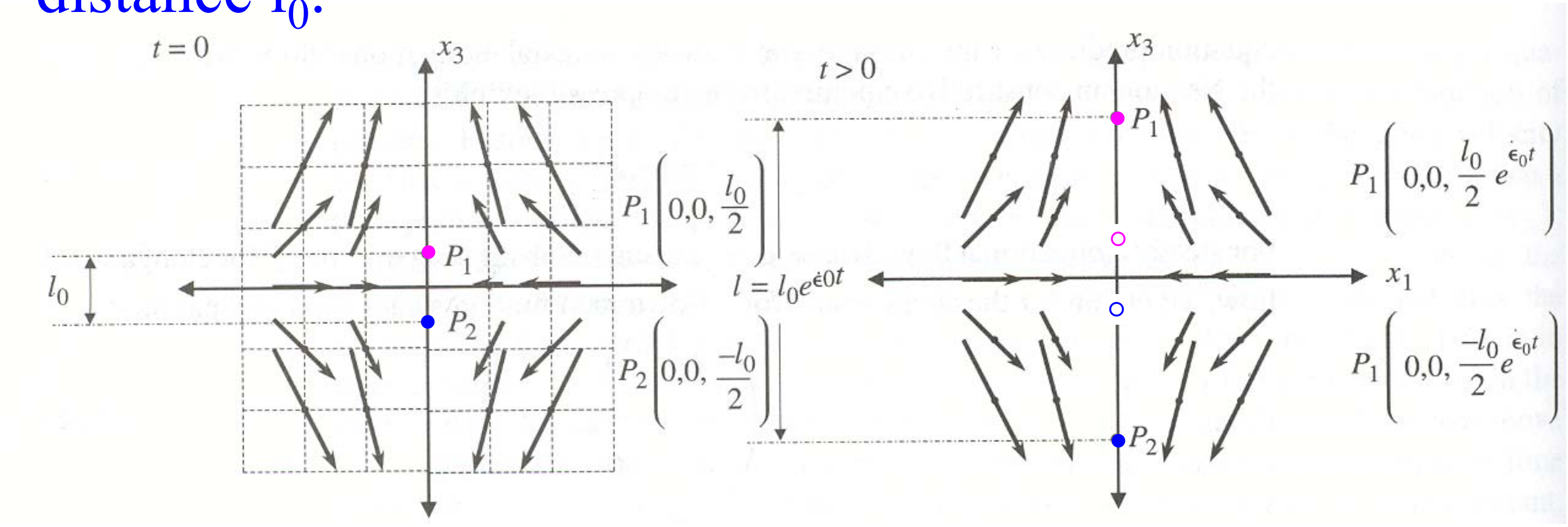
Rate of strain tensor and its magnitude in uniaxial extension

$$\underline{\underline{\dot{\gamma}}} = \begin{pmatrix} -\dot{\varepsilon}(t) & 0 & 0 \\ 0 & -\dot{\varepsilon}(t) & 0 \\ 0 & 0 & 2\dot{\varepsilon}(t) \end{pmatrix}_{123}$$

$$\dot{\gamma} = \left|\underline{\underline{\dot{\gamma}}}\right| = \sqrt{\frac{II_{\dot{\gamma}}}{2}} = \left|\dot{\varepsilon}(t)\right|\sqrt{3}$$

Particle separation in steady uniaxial extensional flow

Two particles on the x_3 axis initially separated by a distance l_0.



Distance between P_1 and P_2 as a function of time: $\ell = \ell_0 \exp(\dot{\varepsilon} t)$

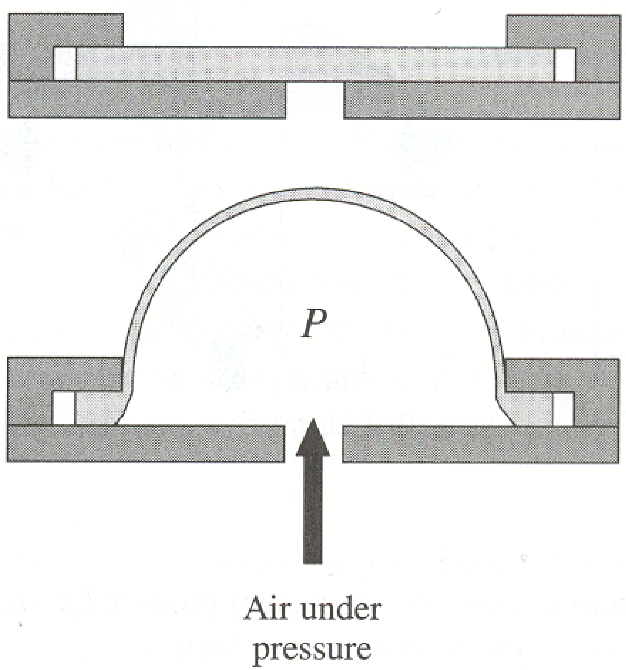
Biaxial elongational flow

Same form of velocity profile as uniaxial stretching flow except $\dot{\varepsilon}(t) < 0$:

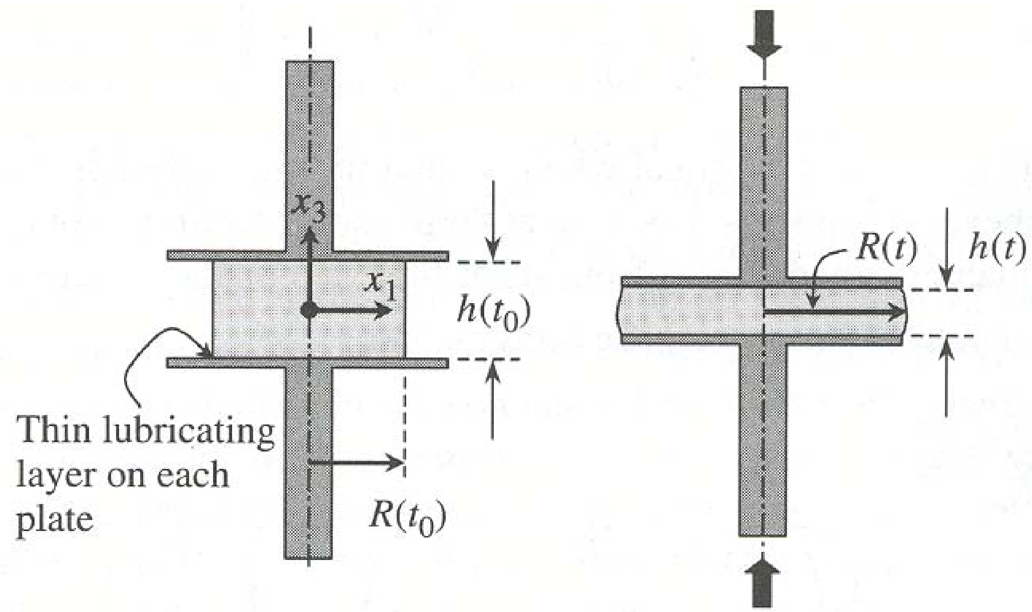
$$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123} = \begin{pmatrix} -\frac{\dot{\varepsilon}(t)}{2} x_1 \\ -\frac{\dot{\varepsilon}(t)}{2} x_2 \\ \dot{\varepsilon}(t) x_3 \end{pmatrix}_{123}$$

where $\dot{\varepsilon}(t) < 0$

Geometries used to produce biaxial extensional flow



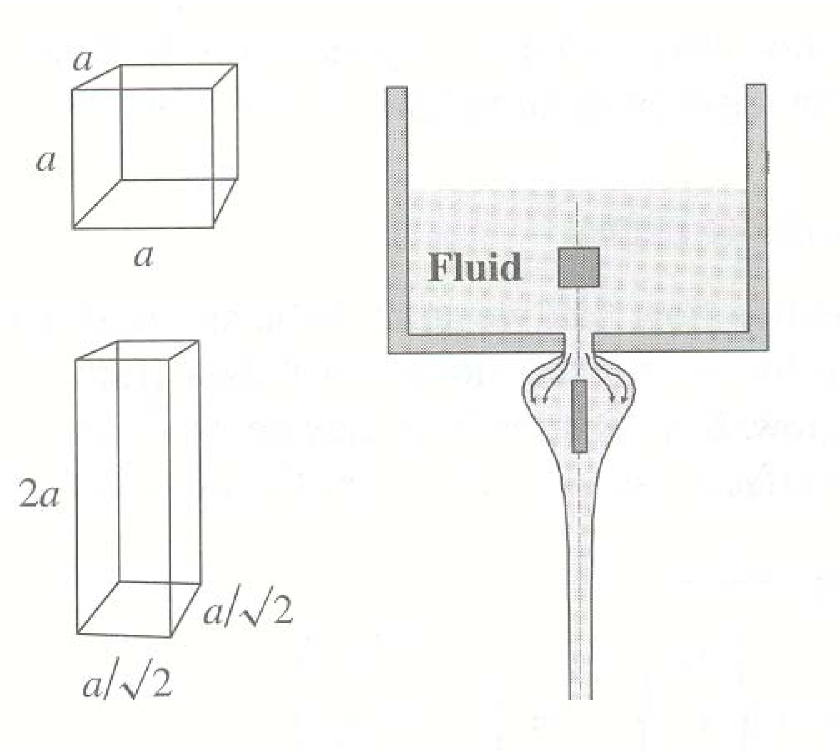
Film inflation



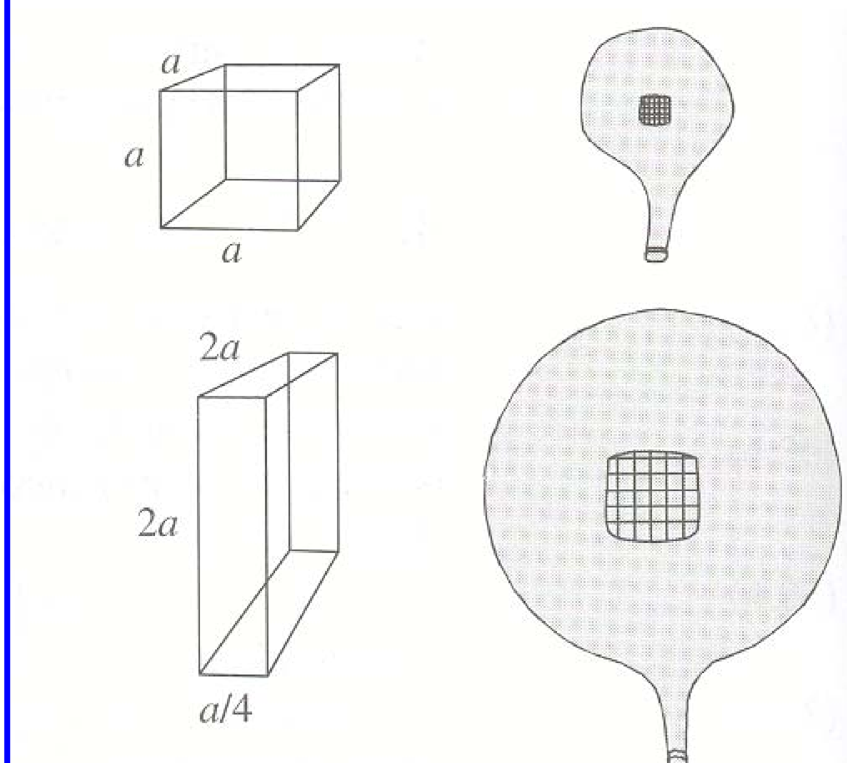
Lubricated squeezing

Comparison between deformation in uniaxial and biaxial extension

Uniaxial extension



Biaxial extension



Since molten polymers are incompressible fluids volume is conserved in the deformation.

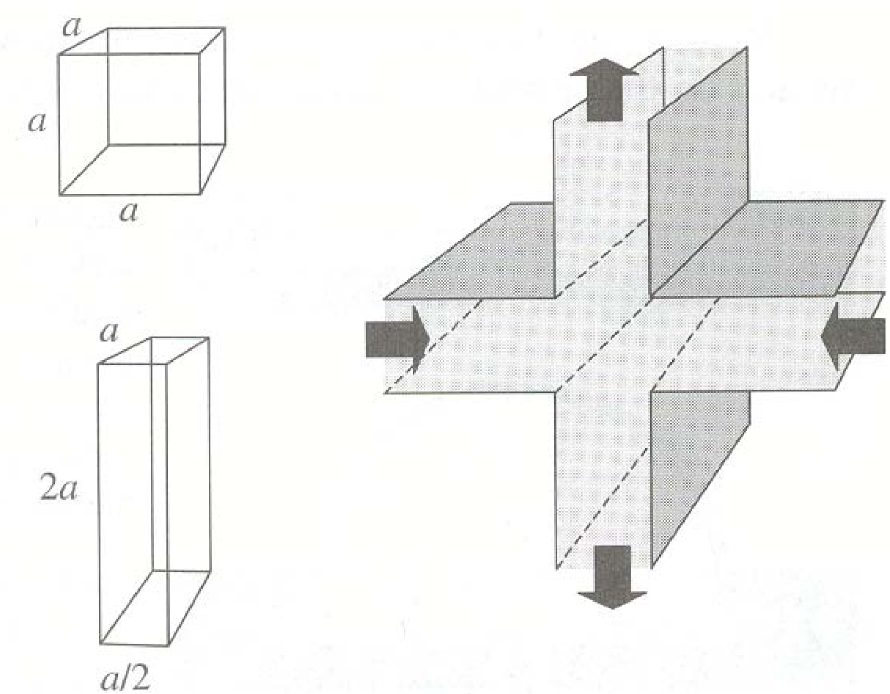
Planar elongational flow

Velocity field:

$$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123} = \begin{pmatrix} -\dot{\varepsilon}(t)x_1 \\ 0 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

where $\dot{\varepsilon}(t) > 0$

Deformation:



General expression for elongational flows

$$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123} = \begin{pmatrix} -\frac{\dot{\varepsilon}(t)}{2}(1+b)x_1 \\ -\frac{\dot{\varepsilon}(t)}{2}(1-b)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

Flow	b	$\dot{\varepsilon}$
Uniaxial	0	>0
Biaxial	0	<0
Planar	1	>0

Stress tensor in elongational flow

Because of symmetry there are only 3 nonzero components of the stress tensor:

$$\underline{\underline{\Pi}} = p\underline{\underline{\delta}} + \underline{\underline{\sigma}} = \begin{pmatrix} p+\sigma_{11} & 0 & 0 \\ 0 & p+\sigma_{22} & 0 \\ 0 & 0 & p+\sigma_{33} \end{pmatrix}$$

Elongational Flow

$$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}_{123} = \begin{pmatrix} -\frac{\dot{\varepsilon}(t)}{2}(1+b)x_1 \\ -\frac{\dot{\varepsilon}(t)}{2}(1-b)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

Vary $\dot{\varepsilon}(t)$ and b to create different flows and measure different material functions. In elongation flows of incompressible fluids we can measure only (σ_{33}- σ_{11}) and (σ_{22}- σ_{11}).

Steady Elongation

Kinematics of steady elongation:

$$\dot{\varepsilon}(t) = \dot{\varepsilon}_0 = \text{constant}$$

Uniaxial:

$$b = 0 \quad \dot{\varepsilon}_0 > 0$$

uniaxial elongational viscosity:

$$\bar{\eta}(\dot{\varepsilon}_0) \equiv \frac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$$

Biaxial:

$$b = 0 \quad \dot{\varepsilon}_0 < 0$$

biaxial elongational viscosity:

$$\bar{\eta}_B(\dot{\varepsilon}_0) \equiv \frac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$$

Planar:

$$b = 1 \quad \dot{\varepsilon}_0 > 0$$

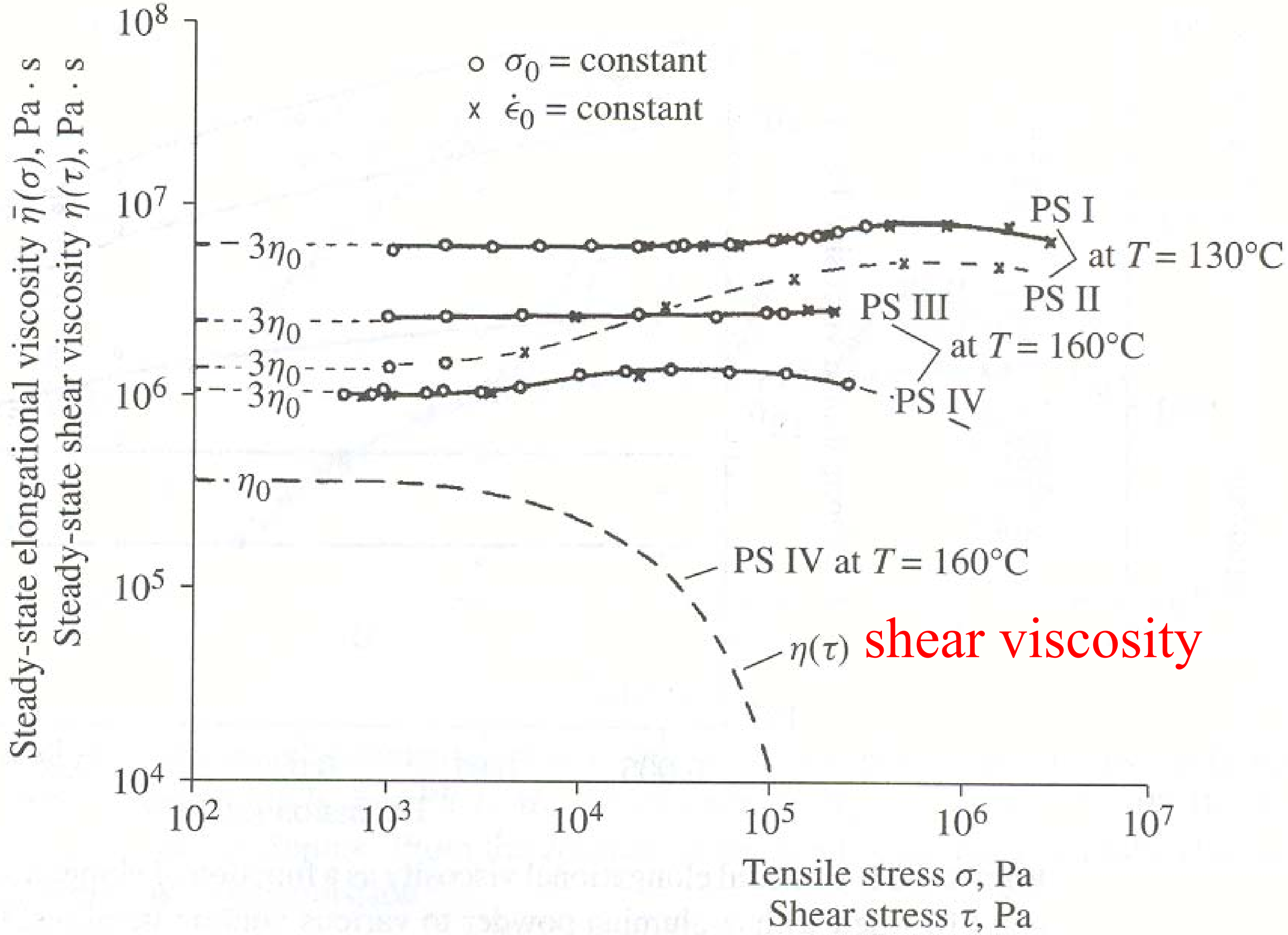
first planar elongational viscosity:

$$\bar{\eta}_{P1}(\dot{\varepsilon}_0) \equiv \frac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$$

second planar elongational viscosity:

$$\bar{\eta}_{P2}(\dot{\varepsilon}_0) \equiv \frac{-(\tau_{22} - \tau_{11})}{\dot{\varepsilon}_0}$$

Steady elongational flow: experimental data



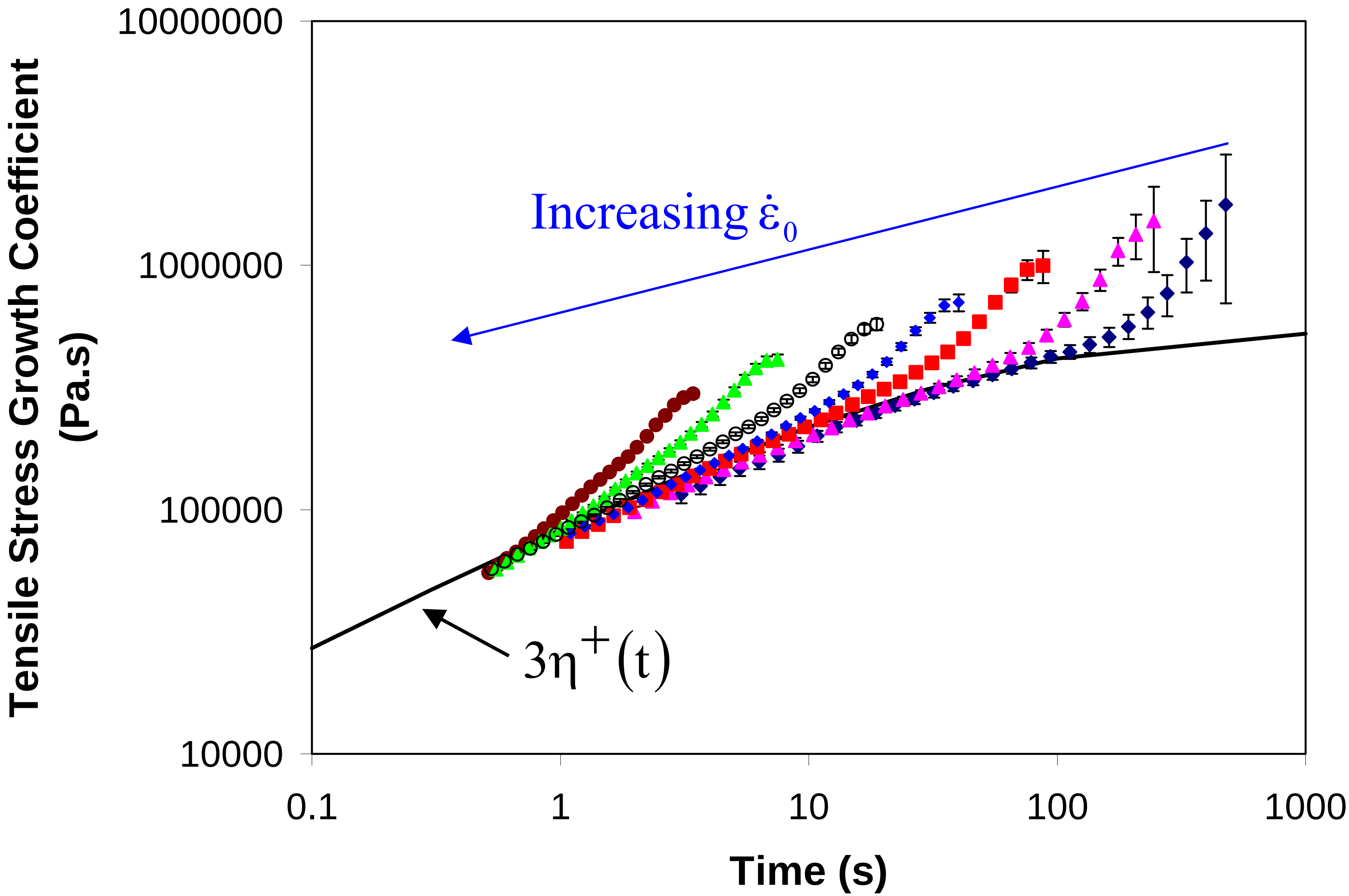
Startup of steady elongational flow

Kinematics of startup of steady elongation:

$$\dot{\varepsilon}(t) = \begin{cases} 0 & t < 0 \\ \dot{\varepsilon}_0 & t \geq 0 \end{cases}$$

Uniaxial elongational stress growth coefficient	$(b = 0, \dot{\epsilon}_0 > 0)$	$\bar{\eta}^+(t, \dot{\epsilon}_0) \equiv \frac{-(\tau_{33} - \tau_{11})}{\dot{\epsilon}_0}$
Biaxial elongational stress growth coefficient	$(b = 0, \dot{\epsilon}_0 < 0)$	$\bar{\eta}_B^+(t, \dot{\epsilon}_0) \equiv \frac{-(\tau_{33} - \tau_{11})}{\dot{\epsilon}_0}$
Planar elongational stress growth coefficients	$(b = 1, \dot{\epsilon}_0 > 0)$	$\bar{\eta}_{P_1}^+(t, \dot{\epsilon}_0) = \bar{\eta}_P^+(t, \dot{\epsilon}_0) \equiv \frac{-(\tau_{33} - \tau_{11})}{\dot{\epsilon}_0}$
		$\bar{\eta}_{P_2}^+(t, \dot{\epsilon}_0) \equiv \frac{-(\tau_{22} - \tau_{11})}{\dot{\epsilon}_0}$

Startup of steady uniaxial elongational flow



Strain in Elongational Flow

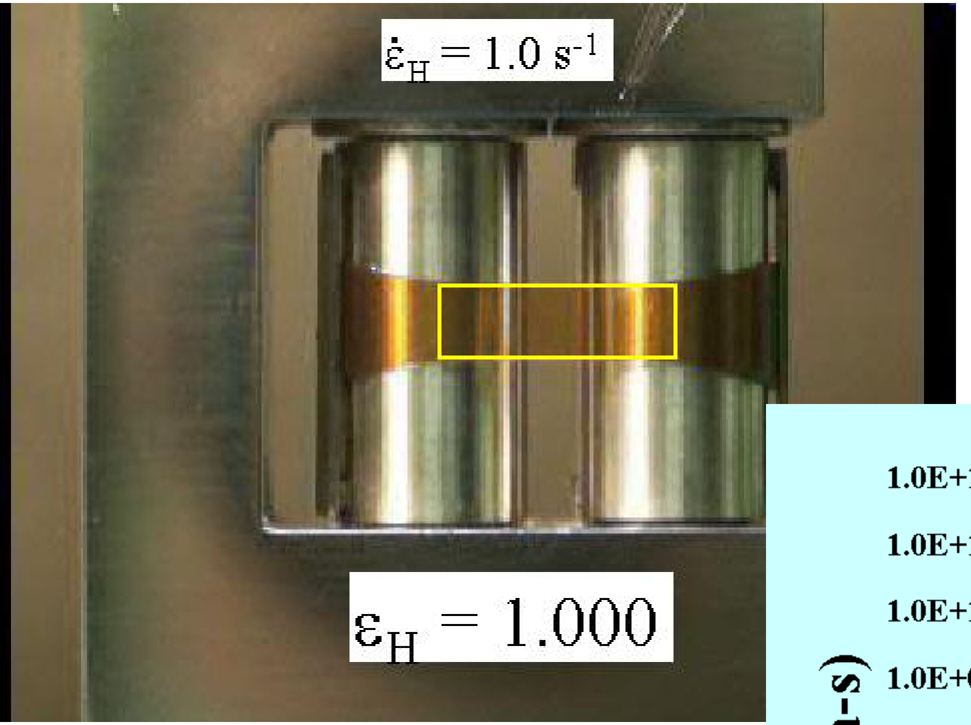
Elongational strain: $\varepsilon(t_{ref}, t) \equiv \int_{t_{ref}}^{t} \dot{\varepsilon}(t')dt'$

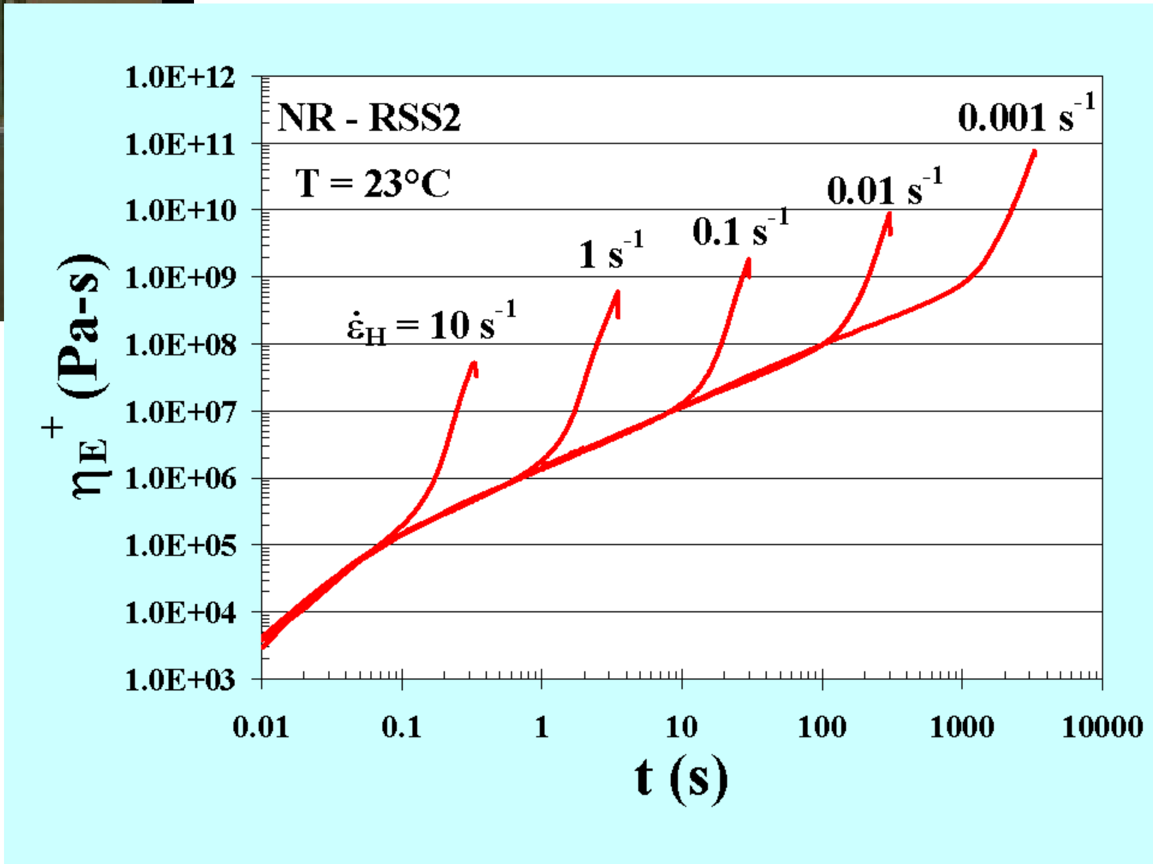
Steady elongational flow,
Hencky Strain:

$$\varepsilon(0, t) = \dot{\varepsilon}_0 t = \ln\left(\frac{\ell(t)}{\ell_0}\right)$$

where ℓ_0 is the initial sample length and $\ell(t)$ is the length at time t.

SER Uniaxial extension rheometer





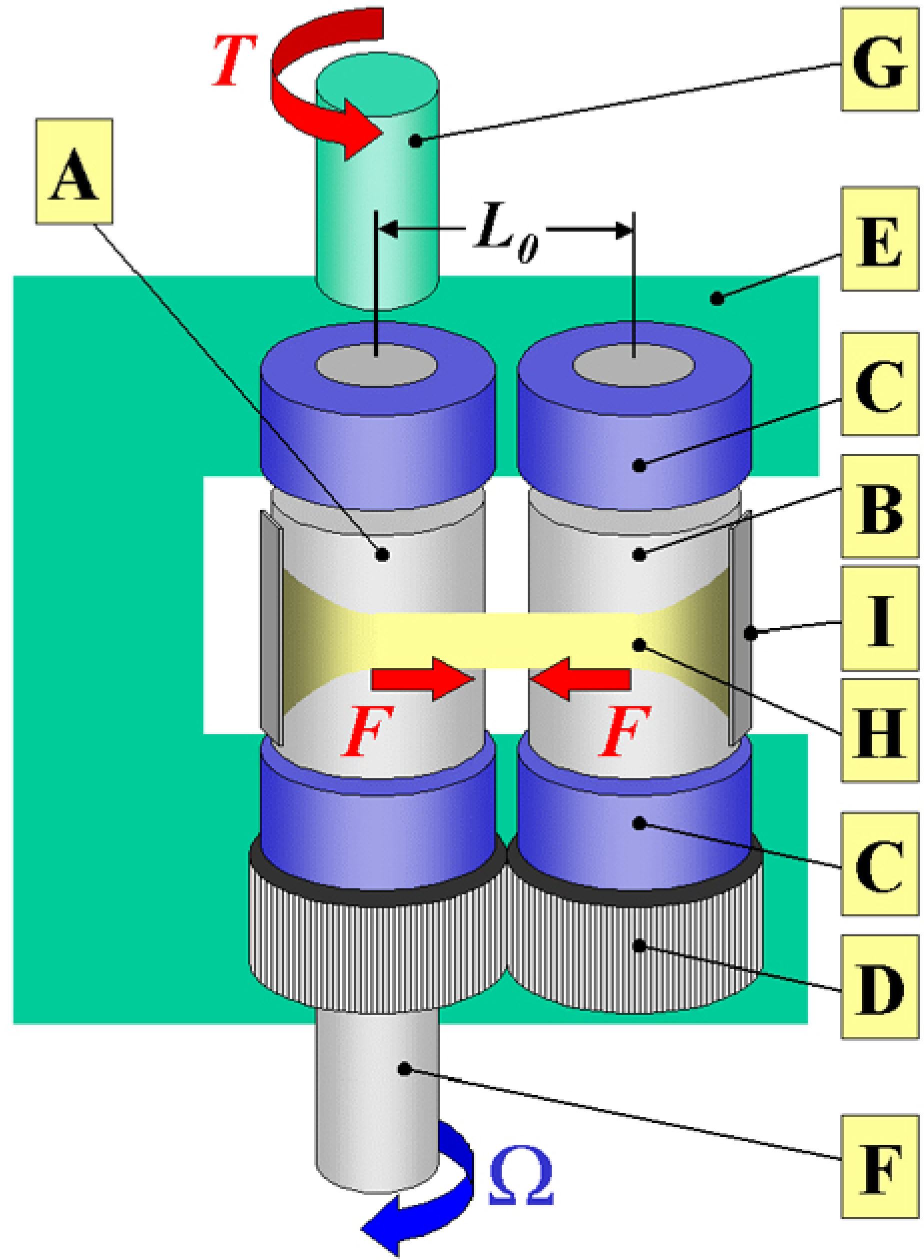
SER Uniaxial extension rheometer

Specifications:

INSTRUMENT PARAMETER	*SPECIFICATION*
Maximum Operating Torque	**2500 g-cm**
Minimum Torque Threshold	**< 0.2 g-cm**
Maximum Recommended Hencky Strain Rate	**20 s^{-1}**
Maximum Hencky Strain Per Drum Revolution	**4**
Operating Temperature Range	**0°C to 250°C**
Windup Drum Diameter	**1.031 cm (0.406 in)**
Stretch Zone Gage Length	**1.272 cm (0.501 in)**
SAMPLE PARAMETER	*SPECIFICATION*
Min. Melt Zero-Shear Viscosity (in Extension Mode)	**~ 10,000 Pa-s**
Sample Mass Range	**5 – 200 mg**
Recommended Sample Width Range	**0.1 – 1.27 cm**
Recommended Sample Thickness Range	**0.005 – 0.1 cm**

SER Uniaxial extension rheometer

- Paired master [A] and slave [B] windup drums
- Housed in bearings [C] within a chassis [E]
- Mechanically coupled via intermeshing gears [D]
- Rotation of the drive shaft [F] results in a rotation of master drum [A] and an equal opposite rotation of slave drum [B]
- Sample [H] secured to the drums by means of securing clamps [I] is wound up onto the drums and stretched over an unsupported length, L_0



SER principle of operation

- For a constant drive shaft rotation rate, W, the Hencky strain rate applied to the sample specimen can be expressed as:

$$d\varepsilon/dt = 2\ \Omega R/L_0$$

with R = radius of the equal dimension windup drums

L_0 = fixed, unsupported length of the specimen sample

- The material's resistance to stretch is manifested as a tangential force, F, acting on the drums and translated as a torque upon the chassis housing the assembly

SER principle of operation

- The resultant torque, T, transmitted through the chassis to the torque shaft [G] is determined from the summation of moments about the axis of the torque shaft:

$$T = 2\,(F + F_F)\,R$$

with T = resultant torque measured by the torque transducer

F_F = frictional contribution from the bearings and intermeshing gears (typically less than 2.5% of the measured torque signal)

- Therefore:

$$T = 2\,F\,R$$

SER principle of operation

- If there is no deviation between the nominal and actual strain rates, the instantaneous cross-sectional area, A(t), of the stretched specimen changes exponentially with time for a constant Hencky strain rate experiment:

$$A(t) = A_0 \exp[-(d\varepsilon/dt)\, t]$$

with A_0 = initial cross-sectional area of the unstretched specimen

SER principle of operation

- For a constant Hencky strain rate, the tensile stress growth function, $\eta_E+(t)$, of the stretched sample is:

$$\eta_E+(t) = F(t) / [A(t) (d\varepsilon/dt)]$$

with F(t) = instantaneous extensional force at time *t* exerted by the sample.

SER Uniaxial extension rheometer