Shear Creep

Impose a constant shear stress and measure strain with time.

At steady state both shear rate and stress are constant, therefore steady state results (i.e. the viscosity curve) are the same whether the flow is shear rate controlled or stress controlled. The transient responses are different however and are therefore described by different material functions.

Kinematics for shear creep:

$$\tau_{21}(t) = \begin{cases} 0 & t < 0 \\ \tau_0 & t \geq 0 \end{cases}$$

Shear Strain

Strain is a measure of shape of a fluid particle at a particular time with respect to its shape at another time.

The definition of strain that we are considering now is only valid in shear flows.

Infinitesimal shear strain:

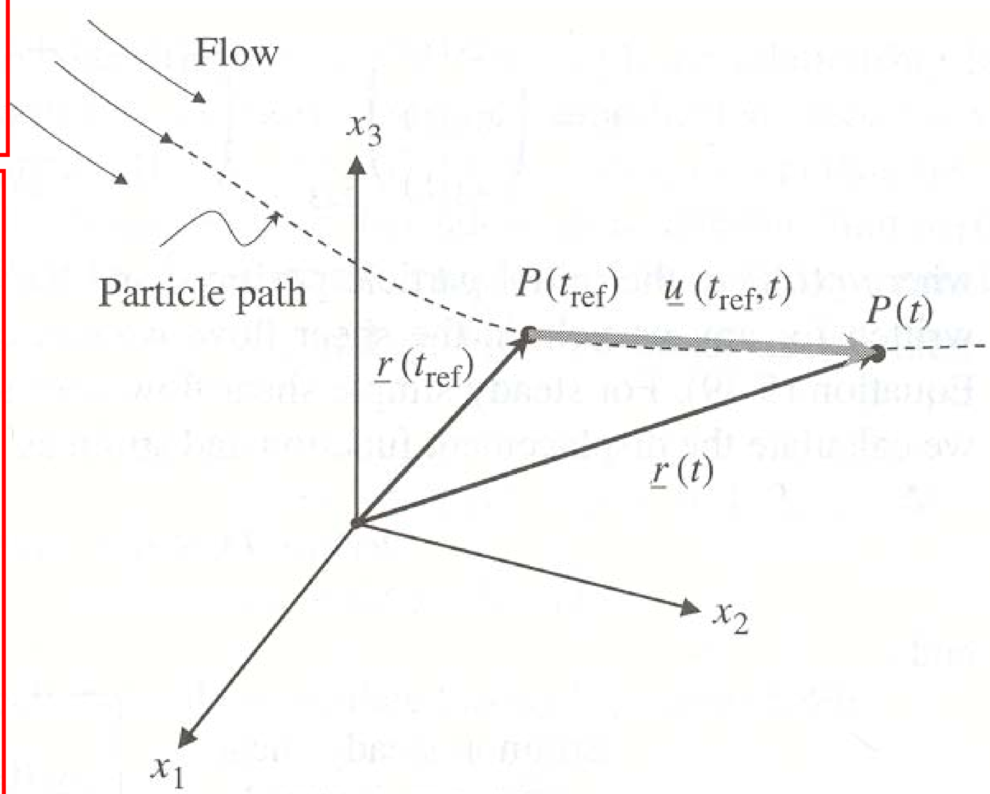
$$\gamma_{21}(t_{ref}, t) = \frac{\partial u_1}{\partial x_2}$$

where

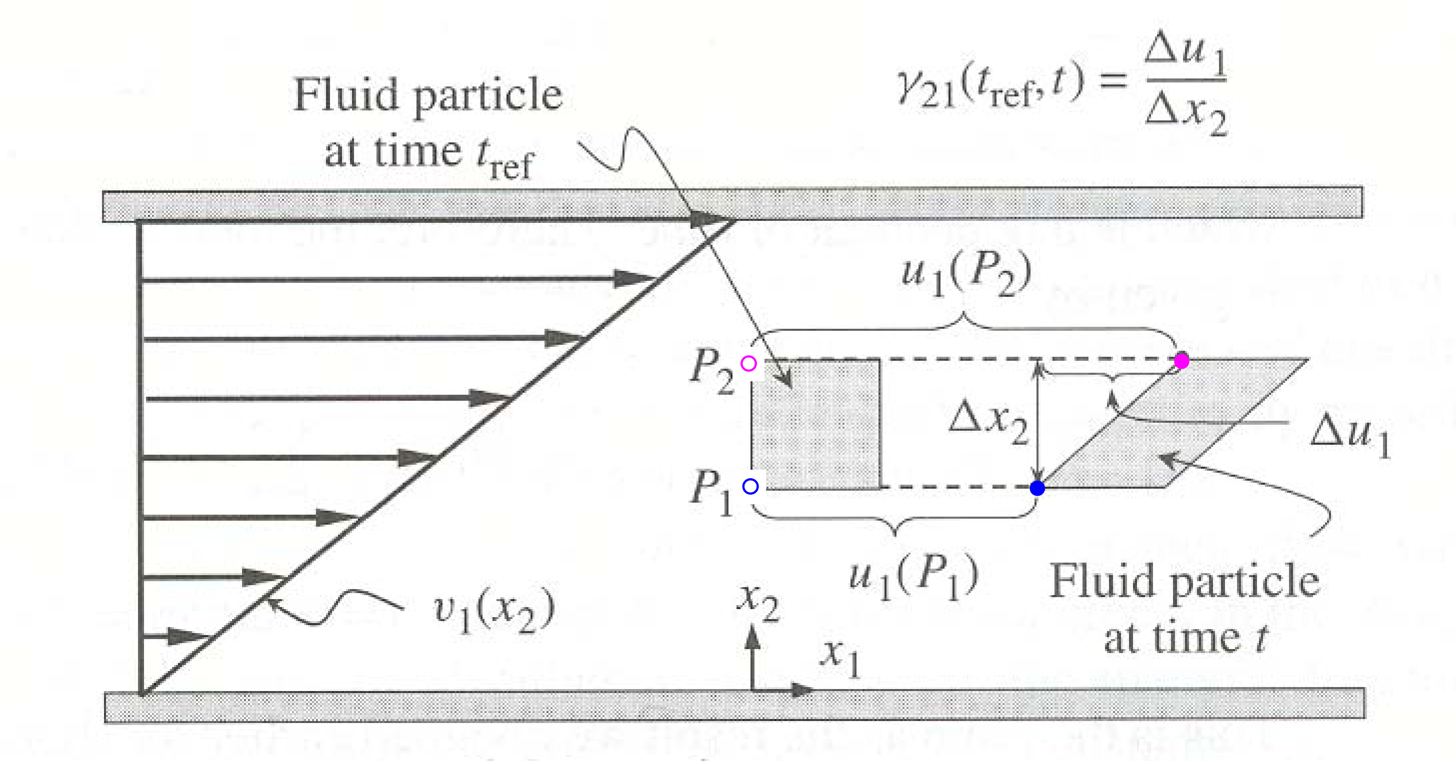
$$u_1 = u_1(t_{ref}, t)$$

and

$$\underline{u} = \underline{r}(t) - \underline{r}(t_{ref}) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}_{123} - \begin{pmatrix} x_1(t_{ref}) \\ x_2(t_{ref}) \\ x_3(t_{ref}) \end{pmatrix}_{123}$$



Shear Strain



Strain in steady and unsteady shear flow

Steady Shear:

$$\gamma_{21}(0,t) = \frac{\partial u_1}{\partial x_2} = \dot{\gamma}_0 t$$

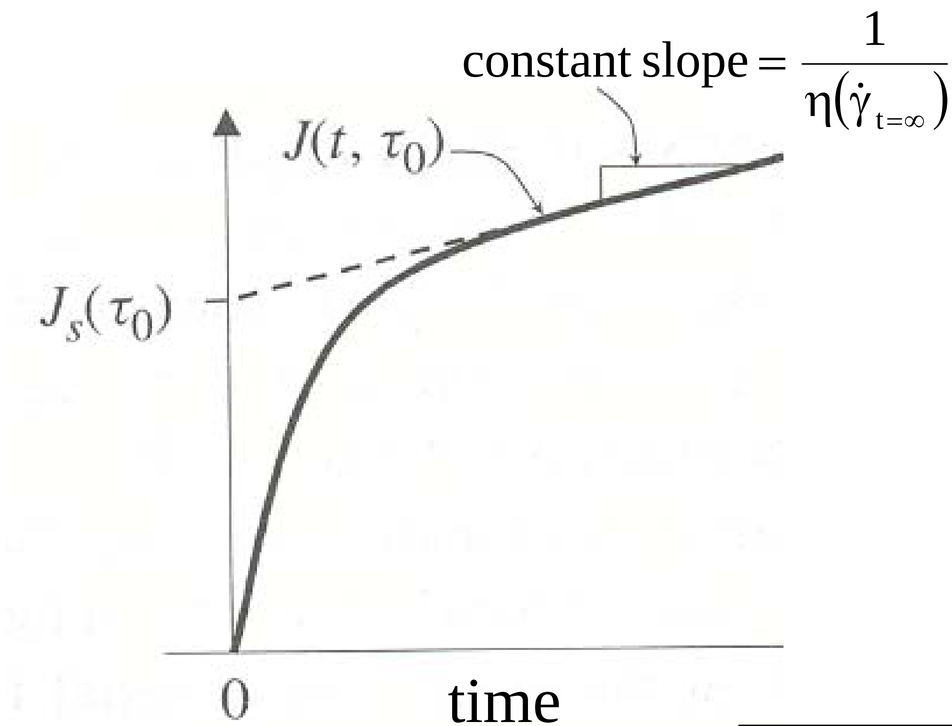
Unsteady Shear:

$$\gamma_{21}(t_1, t_2) = \int_{t_1}^{t_2} \dot{\gamma}_{21}(t')dt'$$

Creep is an unsteady shear flow because the rate of deformation is varying in the initial phase of the flow. In creep measurements the measured variable is strain, i.e. we do not have a functional form of $\dot{\gamma}_{21}(t)$.

Creep Compliance

$$J(t,\tau_0) \equiv \frac{\gamma_{21}(0,t)}{-\tau_0}$$



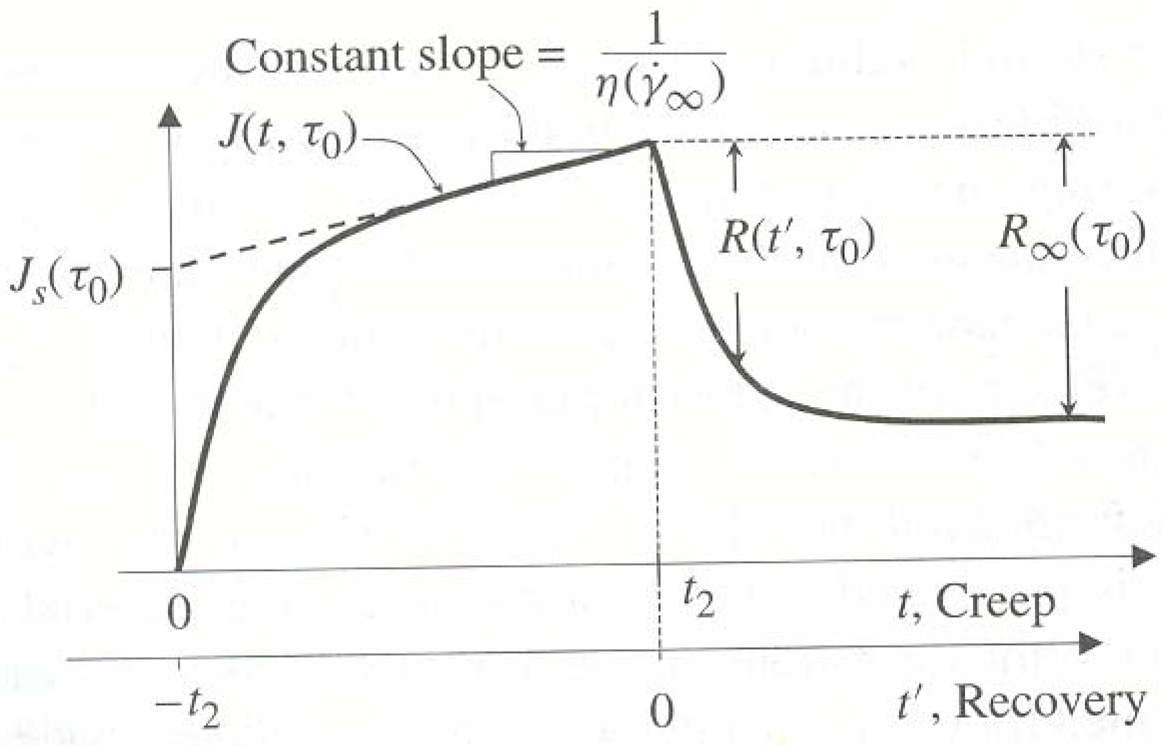
steady state compliance

At steady state:

$$J(t,\tau_0)\Big|_{\text{steady state}} = J_S(\tau_0) + \frac{t}{\eta(\dot{\gamma}_{t=\infty})}$$

Creep Recovery

After a creep deformation has reached steady state then the stress is suddenly removed. Elastic and viscoelastic materials will then recoil back in the opposite direction to the creep flow direction.



Recoil or Recoverable Strain

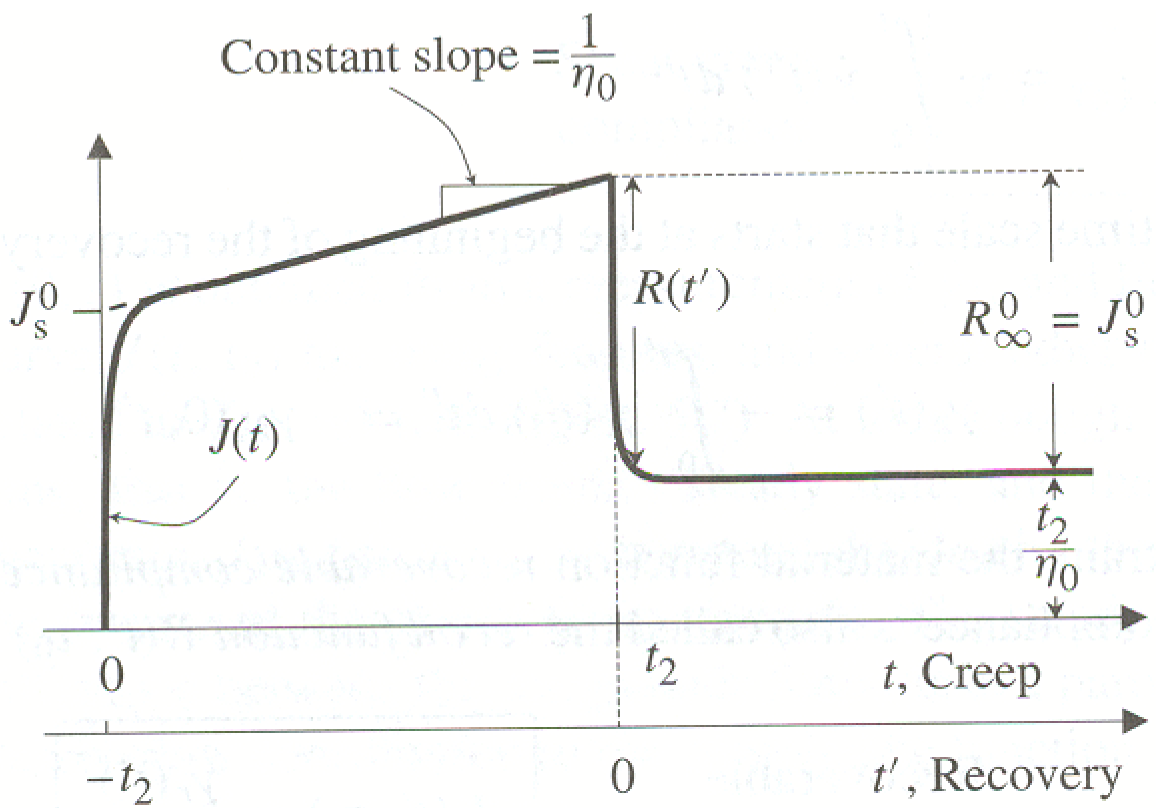
Recoil strain $\gamma_r \equiv \gamma_{21}(0, t_2) - \gamma_{21}(0, t)$

In terms of t $\gamma_r(t) = -\int_{t_2}^{t} \dot{\gamma}(t'')dt''$

In terms of t' $\gamma_r(t') = -\int_{0}^{t'} \dot{\gamma}(t'')dt''$

Recoverable compliance or Recoil function: $J_r(t', \tau_0) = R(t', \tau_0) \equiv \frac{\gamma_r(t')}{-\tau_0}$

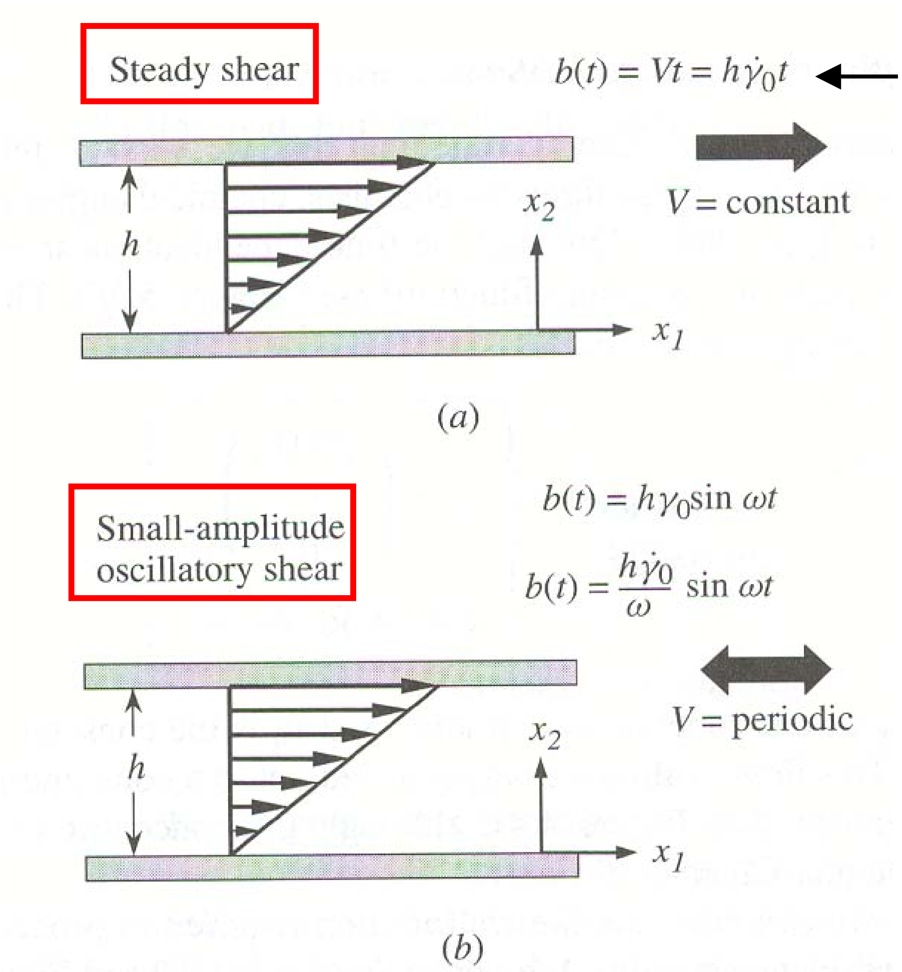
Linear viscoelastic limit of creep and recovery functions



$$J(t) = R(t) + \frac{t}{\eta_0}$$

$$J(t)\Big|_{\text{steady state}} = J_s^0 + \frac{t}{\eta_0}$$

Small amplitude oscillatory shear



← plate position

We call this flow small amplitude oscillatory shear because the amplitude is small enough such that the material's responses is linear viscoelastic.

Small amplitude oscillatory shear

Strain: $$\gamma_{21}(0,t) = \frac{\dot{\gamma}_0}{\omega} \sin \omega t = \gamma_0 \sin \omega t$$

phase shift

Stress: $$-\tau_{21}(t) = \tau_0 \sin(\omega t + \delta)$$
$$= (\tau_0 \cos \delta) \sin \omega t + (\tau_0 \sin \delta) \cos \omega t$$

in phase with the imposed strain

out of phase with the imposed strain

Small amplitude oscillatory shear

Newtonian fluid:

$$\tau_{21} = -\mu\dot{\gamma}_{21} = -\mu\dot{\gamma}_0 \cos\omega t$$

completely out-of-phase with the strain

Elastic solid:

$$\tau_{21} = -G\gamma_{21} = -G\gamma_0 \sin\omega t$$

completely in-phase with the strain

For a viscoelastic fluid the out-of-phase portion of the stress response in SAOS corresponds to the viscous response of the fluid and the in-phase portion corresponds to the elastic storage of energy.

Complex modulus

$$-\tau_{21}(t) = \gamma_0 G' \sin \omega t + \gamma_0 G'' \cos \omega t$$

Storage modulus: $G' \equiv \frac{\tau_0}{\gamma_0} \cos \delta$ elastic response, in phase with strain

Loss modulus: $G'' \equiv \frac{\tau_0}{\gamma_0} \sin \delta$ viscous response, out of phase with the strain.

Complex notation: $G^*(\omega) = G'(\omega) + iG''(\omega)$

↑ complex modulus

Complex viscosity

$$-\tau_{21}(t) = \dot{\gamma}_0 \eta' \cos \omega t + \dot{\gamma}_0 \eta'' \sin \omega t$$

$$\eta' \equiv \frac{\tau_0}{\dot{\gamma}_0} \sin \delta = \frac{G''}{\omega}$$

$$\eta'' \equiv \frac{\tau_0}{\dot{\gamma}_0} \cos \delta = \frac{G'}{\omega}$$

Complex notation: $\eta^*(\omega) = \eta'(\omega) - i\eta''(\omega)$

Other SAOS material functions

TABLE 5.1
Definitions of Material Functions for Small-Amplitude Oscillatory Shear (SAOS) in Terms of Storage Modulus G' and Loss Modulus G''

Complex modulus magnitude	$\lvert G^*\rvert = \sqrt{G'^2 + G''^2}$
Loss tangent	$\tan\delta = \frac{G''}{G'}$
Dynamic viscosity	$\eta' = \frac{G''}{\omega}$
Out-of-phase component of η^*	$\eta'' = \frac{G'}{\omega}$
Complex viscosity magnitude	$\lvert\eta^*\rvert = \sqrt{\eta'^2 + \eta''^2}$
Complex compliance magnitude	$\lvert J^*\rvert = \frac{1}{\lvert G^*\rvert}$
Storage compliance	$J' = \frac{1/G'}{1+\tan^2\delta}$
Loss compliance	$J'' = \frac{1/G''}{1+(\tan^2\delta)^{-1}}$