Example: Theory of LVE

- Derive an expression for the uniaxial elongational stress growth coefficient for N maxwell elements in parallel.
- Using the previous result, give an expression for the uniaxial elongational viscosity.

(1) $G(t) = \sum_{i=1}^{N} G_i \exp(-t/\lambda_i)$

startup of uniaxial extension: $\dot{\epsilon}_0$ = Hencky strain rate

$$\dot{\gamma}_{33} = 2\dot{\epsilon}_0 \quad t \geq 0$$

$$\dot{\gamma}_{11} = -\dot{\epsilon}_0 \quad t \geq 0$$

$$\eta_E^+(t) = \frac{-\{\tau_{33}(t) - \tau_{11}(t)\}}{\dot{\epsilon}_0}$$

$$\tau_{ii} = -\int_{-\infty}^{t} G(t-t')\dot{\gamma}_{ii}(t')\,dt'$$

note: for $t' < 0$ $\dot{\gamma}_{ij} = 0$

$$\tau_{33} = -\int_0^t G(t)(2\dot{\epsilon}_0)\,dt'$$

sub in for this.

$$\tau_{33} = -\int_0^t \sum G_i \exp\left(-\frac{(t-t')}{\lambda_i}\right)(2\dot{\epsilon}_0)\,dt'$$

Bring constants outside integral

$$\tau_{33} = -2\dot{\epsilon}_0 \sum G_i \int_0^t \exp\left[-\frac{(t-t')}{\lambda_i}\right]dt'$$

$$\tau_{33} = -2\dot{\epsilon}_0 \sum G_i \lambda_i \exp\left[-\frac{(t-t')}{\lambda_i}\right]_0^t$$

$$\tau_{33} = -2\dot{\epsilon}_0 \sum G_i \lambda_i \left[1 - \exp(-t/\lambda_i)\right]$$

similarly: $\tau_{11} = \dot{\epsilon}_0 \sum G_i \lambda_i \left[1 - \exp\left(-\frac{t}{\lambda_i}\right)\right]$

then:

$$\eta_E^+(t) = \frac{-\left[-2\dot{\epsilon}_0 - \dot{\epsilon}_0\right]}{\dot{\epsilon}_0} \sum G_i \lambda_i \left[1 - \exp(-t/\lambda_i)\right]$$

$$\eta_E^+(t) = 3 \sum G_i \lambda_i \left[1 - \exp(-t/\lambda_i)\right]$$

(2) Uniaxial elongational viscosity

$$\eta_E = \lim_{t \to \infty} \eta_E^+(t)$$

$$\eta_E = \lim_{t \to \infty} 3 \sum G_i \lambda_i \left[1 - \exp(-t/\lambda_i)\right]$$

$$\eta_E = 3 \sum G_i \lambda_i$$